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THE DISTRIBUTION OF THE PRESSURE OF THE DISPERSED PHASE ALONG A SURFACE OF A BUBBLE WITH AN IRREGULAR LEADING STAGNATION POINT†

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The plane problem of a deformed bubble rising in a fluidized bed is considered. The bubble is represented by a reflection of a circular segment in a vertical axis.

LET a_b BE the radius of the circular segment and $2c$ the length of the part of the axis of symmetry that belongs to the bubble. We will use a system of bipolar coordinates (ξ, η): $\xi=\theta_1-\theta_2$, $\eta=\ln(r_2/r_1)$, attached to the bubble, where θ_i and r_i $(i=1, 2)$ are the polar angles and radii of the current point, respectively (Fig. 1).

The equation of the bubble surface in (ξ, η) coordinates has the form $\xi=\pi n/2$, and the leading stagnation point corresponds to $\eta=-\infty$. The parameter $n\in[0, 2]$ characterizes the degree of deformation of the bubble's cavity: the cavities are shaped like an apple for $n\in(0, 1)$ and like a lens for $n\in(1, 2)$. The case $n=0$ corresponds to a cavity in a form of two circular bubbles in contact with each other (or to a bubble touching a vertical wall), for $n=1$ there is no deformation of a cavity, and we have a single circular bubble, and for $n=2$ and $a_b\to\infty$ the cavity degenerates to a slit of length $2c$ parallel to the flow direction.

Within a framework of Davidson's model the motion of particles around a bubble in a fluidized bed is identical with the irrotational flow of an ideal fluid with overall pressure $p=p_f+p_s$, where p_f is the pressure in the fluid phase and p_s is the effective pressure in the solid phase [1]. The stream function of this flow has the form [2]

$$\psi_s=\frac{2\delta}{n}\frac{\sin(2\xi/n)U_b a_b}{\operatorname{ch}(2\eta/n)-\cos(2\xi/n)}$$

Here $\delta=c/a_b=\sin(\pi n/2)$ and U_b is the terminal rising velocity of the bubble.

The boundary conditions for the pressure in both phases at the bubble surface are as follows:

$$p_f=p_b(t),\quad p_s=0$$

where p_b is the fluid-phase pressure everywhere in the interior of the bubble. Taking this into account we will write the Bernoulli integral for the leading stagnation point A_0 and for a near-by point A on the cavity surface

$$p_{s0}/\rho d_s+gc=p_s/\rho d_s+gz+w_s^2/2$$

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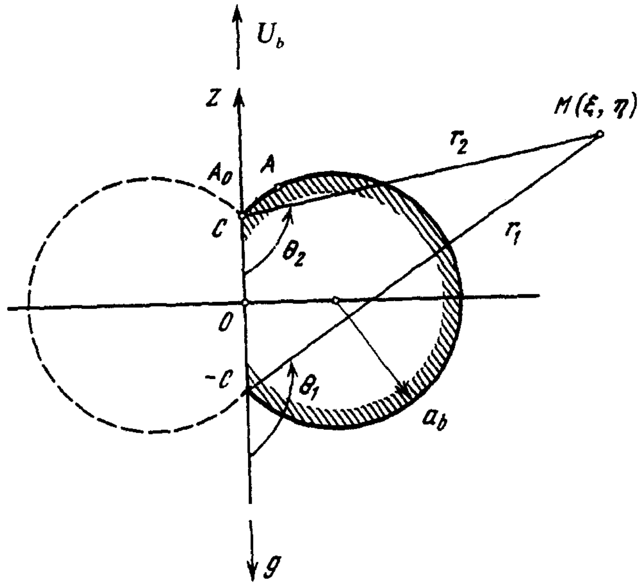

FIG. 1.

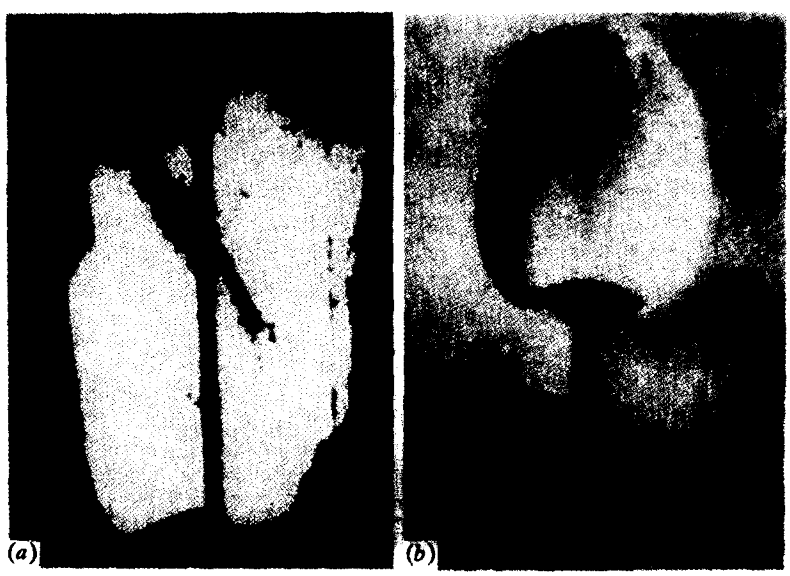

FIG. 2.

where d_s is the density of dispersed particles, ρ is the volume concentration of the solid phase, g is the acceleration due to gravity, z is the vertical coordinate of the point A, measured from the equator in a direction opposite to g, and w_s is the velocity distribution of the particles along the cavity surface. This distribution has the form [2]

$$w_s = \frac{4U_b}{n^2} \frac{\operatorname{ch}\eta - \cos(\pi n/2)}{\operatorname{ch}(2\eta/n)+1}$$

In addition

$$c - z = \frac{c[e^{\eta} - \cos(\pi n/2)]}{\operatorname{ch}\eta - \cos(\pi n/2)}$$

By virtue of the latter two equations the Bernoulli integral is transformed into the dimensionless form

$$p(\eta)=\frac{1}{\mathrm{Fr}}\frac{\sin(\pi n/2)[e^{\eta}-\cos(\pi n/2)]}{\mathrm{ch}\,\eta-\cos(\pi n/2)}-\frac{2}{n^4}\frac{[\mathrm{ch}\,\eta-\cos(\pi n/2)]^2}{\mathrm{ch}^4(\eta/n)}$$

Here $p(\eta)=(p_s-p_{s0})/d_s\rho U_b^2$, $\mathrm{Fr}=U_b^2/ga_b$ is the Froude number, and $p(\eta)\to 0$ as $A\to A_0$ $(\eta\to-\infty)$.

The estimate of the bubble rising velocity by the Davis–Taylor method [3] is based on the assumption that the bubble surface (at least in the neighbourhood of the leading stagnation point) coincides with the line of constant pressure of the dispersed phase $(\rho(\eta)\equiv 0)$, and both terms on the right-hand side of the expression for $p(\eta)$ have the same order of smallness as $\eta\to-\infty$. In fact, letting $n=1$ we conclude that both terms are of the order of $e^{2\eta}$, as $\eta\to\infty$, which yields $\mathrm{Fr}=\frac{1}{4}$ corresponding to the familiar Davis–Taylor formula for the rising velocity of a bubble with circular front

$$U_b=(\tfrac{1}{2})(ga_b)^{\frac{1}{2}}$$

For $n\neq 1$ we have

$$\frac{1}{\mathrm{Fr}}\frac{\sin(\pi n/2)[e^{\eta}-\cos(\pi n/2)]}{\mathrm{ch}\,\eta-\cos(\pi n/2)}\sim e^{\eta},\quad \eta\to-\infty$$

$$\frac{2}{n^4}\frac{[\mathrm{ch}\,\eta-\cos(\pi n/2)]^2}{\mathrm{ch}^4(\eta/n)}\sim e^{-2\eta(1-2/n)},\quad \eta\to-\infty$$

For $n=\frac{4}{3}$ we have $-2\eta(1-2/n)=\eta$, so that $U_b=(\frac{4}{3}^{7/4})(ga_b)^{1/2}$, and $2c/a_b=\sqrt{3}$ corresponds to a contour shaped like a lens elongated in the flow direction.

For $n\neq 1$ and $n\neq\frac{4}{3}$ the above procedure for evaluating U_b for the bubble shapes considered does not hold. For instance, for $n\in(0,\,1)$ the bubble surface in the neighbourhood of the leading stagnation point cannot be a line where the solid phase pressure is constant, which is a necessary condition for the steady rising of a bubble; in this case, the right-hand side of the formula for $p(\eta)$ is essentially negative.

From the arguments given above it follows that for $n\neq 1$ and $n\neq\frac{4}{3}$ the rising of a bubble with an irregular leading stagnation point is unsteady in principle. This is confirmed by the observations of so-called "fingers" that occur at the bubble surface in the neighbourhood of the leading stagnation point. The evolution of such "fingers" often leads to breakdown of a rising bubble [3].

On the other hand, the existence of a bubble shape with a stable front for $n=\frac{4}{3}$ may explain the typical elongation of flat bubbles (or bubbles at the wall) in a fluidized bed; the vertical dimensions of such a bubble is often twice the horizontal dimensions [3] (Fig. 2). The presence of a cusp at the leading stagnation point at the cavity surface does not imply any essential drawback of the model, since, by virtue of the boundedness of w_s everywhere at the cavity surface, the flow pattern in the neighbourhood of the leading stagnation point is modified only slightly after minor "regularizing" smoothing of the contour.

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